## Proving Triangles Congruent



| Topic | Pages in Packet | Assignment: <br> (Honors TXTBK) |
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| Identifying Congruent Triangles | Pages 7-13 | This Packet pages 14- 15 |
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## Day 1

## SWBAT: Use properties of congruent triangles. Prove triangles congruent by using the definition of congruence.

Vocabulary Review: Describe how to classify triangles by sides or angles. Draw a diagram for each.

| By Angles | By Sides |
| :--- | :--- |
| Acute | Scalene |
| Right | Isosceles |
| Obtuse |  |
| Equilateral |  |

Theorem Review: Describe each theorem and include a diagram

| Theorem | Diagram |
| :--- | :--- |
| Triangle Sum Theorem |  |
| Exterior Angle Theorem |  |
|  |  |
| Third Angles Theorem |  |

1. Classify the triangles based on their side lengths and angle measures.

$\qquad$
$\qquad$
$\qquad$
$\qquad$

For \#2-5, solve for the variable and then find missing angle or side lengths.
2. If $\mathrm{m} \measuredangle \mathrm{A}=(5 \mathrm{x}-7)^{\circ}$, find $\mathrm{x}, \mathrm{m} \measuredangle \mathrm{A}$ and $\mathrm{m} \measuredangle \mathrm{E}$.

2. $x=$ $\qquad$
$m \measuredangle A=$ $\qquad$
$m \measuredangle E=$ $\qquad$
3. In $\triangle \mathrm{WIN} \mathrm{m} \measuredangle \mathrm{W}=(2 \mathrm{y}+7)^{\circ}, \mathrm{m} \measuredangle \mathrm{I}=(6 \mathrm{y})^{\circ}, \mathrm{m} \measuredangle \mathrm{N}=(8 \mathrm{y}+13)^{\circ}$. Find y .
(Hint: Draw a diagram.)

4. $x=$ $\qquad$
$m \measuredangle A B C=$ $\qquad$
5. In right triangle $A B C, \mathrm{~m} \angle C=3 y-10$, $\mathrm{m} \angle B=y+40$, and $\mathrm{m} \angle A=90$. What type of right triangle is triangle $A B C$ ?
6. The angle measures of a triangle are in the ratio of $5: 6: 7$. Find the angle measures of the triangle.

## 7. Solve for $m \Varangle P R S$



## Challenge

Find the measure of the angle indicated.


## SUMMARY

| Classification | Description |
| :--- | :--- |
| acute triangle | triangle that has three <br> acute angles |
| equiangular triangle | triangle that has three <br> congruent acute angles |
| right triangle | triangle that has one <br> right angle |
| obtuse triangle | triangle that has one <br> obtuse angle |
| equilateral triangle | triangle with three <br> congruent sides |
| isosceles triangle | triangle that has at least <br> two congruent sides |
| scalene triangle | triangle that has no <br> congruent sides |

## Exit Ticket

1. In $\triangle A B C, \mathrm{~m} \angle A=x, \mathrm{~m} \angle B=2 x+2$, and $\mathrm{m} \angle C=3 x+4$. What is the value of $x$ ?
1) 29
2) 31
3) 59
4) 61
2. Triangle $P Q R$ has angles in the ratio of $2: 3: 5$. Which type of triangle is $\triangle P Q R$ ?
1) acute
2) isosceles
3) obtuse
4) right

## Day 2 - Identifying Congruent Triangles

Warm - Up
Find the measure of the missing angles


$$
m \measuredangle 1=
$$

$$
m \measuredangle 2=
$$

$\qquad$

$$
m \not x 3=
$$

$\qquad$
$m \measuredangle 4=$ $\qquad$
$m \measuredangle 5=$ $\qquad$

$$
\begin{aligned}
& m \measuredangle 6= \\
& m \measuredangle 7=
\end{aligned}
$$

Geometric figures are congruent if they are the same size and shape. Corresponding angles and corresponding sides are in the same $\qquad$ in polygons with an equal number of $\qquad$ .

Two polygons are $\qquad$ polygons if and only if their $\qquad$ sides are $\qquad$ .
Thus triangles that are the same size and shape are congruent.

Ex 1: Name all the corresponding sides and angles below if the polygons are congruent.


Corresponding Sides
Corresponding Angles

Ex 2:
Given $\triangle G E O \cong \triangle F U N$. Let $m \measuredangle E=(3 x-4)^{\circ}, m \measuredangle F=2 x^{\circ}, m \measuredangle N=(20-x)^{\circ}$. b.
a. Draw and label a diagram.
b. List all six pairs of congruent parts
c. Solve for x
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
c. $\mathrm{x}=$ $\qquad$

## Identifying Congruent Triangles

| Side-Side-Side (SSS) Congruence Postulate |  |  |
| :--- | :---: | :---: |
| If three sides of one triangle are congruent to three sides |  |  |
| of another triangle, then the triangles are congruent. |  |  |
| $\overline{Q R} \cong \overline{T U}, \overline{R P} \cong \overline{U S}$, and $\overline{P Q} \cong \overline{S T}$, so $\triangle P Q R \cong \triangle S T U$. |  |  |

You can use SSS to explain why $\triangle F J H \cong \triangle F G H$. It is given that $\overline{F J} \cong \overline{F G}$ and that $\overline{J H} \cong \overline{G H}$. By the Reflex. Prop. of $\cong, F H \cong \overline{F H}$. So $\triangle F J H \cong \triangle F G H$ by SSS.


Side-Angle-Side (SAS) Congruence Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\angle N$ is the included angle of $\overline{L N}$ and $\overline{N M}$.
$\triangle H J K \cong \triangle L M N$

## Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

$\triangle A B C \cong \triangle D E F$


## An included side is the common side of two

 consecutive angles in a polygon. The following postulate uses the idea of an included side.
$\overline{P Q}$ is the included side of $\angle P$ and $\angle Q$.

Name the postulate or theorem you would use to prove $\triangle A C B \cong \triangle Z X Y$ given following information. If there is not enough information, state none.

| $\begin{aligned} & \angle B \cong \angle Y \\ & \angle A \cong \angle Z \\ & B C \cong Y X \end{aligned}$ | $\begin{aligned} & \angle C \cong \angle X \\ & \angle A \cong \angle Z \\ & \overline{C A} \cong \overline{X Z} \end{aligned}$ | $\begin{aligned} & \overline{A C} \cong \overline{Z X} \\ & \angle B \cong \angle Y \\ & \overline{B C} \cong \overline{Y X} \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{aligned} \overline{Z X} & \approx \overline{A C} \\ \overline{X^{V}} & \approx \overline{C B} \\ \angle X & \approx \angle C \end{aligned}$ | $\begin{aligned} & \overline{A B} \cong \overline{Z Y} \\ & \overline{A C} \cong \overline{Z X} \\ & \overline{C B} \cong \overline{X Y} \end{aligned}$ | $\overline{A B} \cong \overline{B C},$ <br> $\overline{B D}$ bisects $\angle A B C$. |

The pair of triangles below has two corresponding parts marked as congruent.

1. What additional information is needed for a SAS congruence correspondence?


Answer: $\qquad$ $\cong$ $\qquad$
2.

What additional information is needed for an
ASA congruence correspondence?


Answer: $\qquad$ $\cong$ $\qquad$
3. What additional information is needed for an AAS congruence correspondence?


Answer: $\qquad$ $\cong$
$\qquad$
4. What additional information is needed for a SSS congruence correspondence?


Answer: $\qquad$ $\cong$ $\qquad$
5. What additional information is needed for a
SAS congruence correspondence.


Answer: $\qquad$ $\cong$ $\qquad$

What additional information is needed for an ASA congruence correspondence?


Answer: $\qquad$
$\qquad$

Using the tick marks for each pair of triangles, name the method \{SSS, SAS, ASA, AAS\} that can be used to prove the triangles congruent. If not, write not possible.
(Hint: Remember to look for the reflexive side and vertical angles!!!!)


## Challenge

## Solve for $x$.



## SUMMARY

## Side-Side-Side (S.S.S.)



## Side-Angle-Side

(S.A.S.)


Angle-Side-Angle (A.S.A.)


## Angle-Angle-Side

 (A.A.S.)

## Exit Ticket

As shown in the diagram below, $\overline{A C}$ bisects $\angle B A D$ and $\angle B \cong \angle D$.


Which method could be used to prove
$\triangle A B C \cong \triangle A D C$ ?

1) SSS
2) AAA
3) SAS
4) AAS

Kuta Software - Infinite Geometry
SSS, SAS, ASA, and AAS Congruence

Name $\qquad$ Date $\qquad$ Period $\qquad$
State if the two triangles are congruent. If they are, state how you know.
1)

2)

4)
5)

6)

7)


8)

9)

10)


State what additional information is required in order to know that the triangles are congruent for the reason given.
11) ASA
12) SAS

13) SAS

15) SAS

17) SSS

14) ASA

16) ASA

18) SAS


## Day 3 - Proving Congruent Triangles

## Warm - Up

Each pair of triangles below has two corresponding sides or angles marked congruent. Indicate the additional information needed to enable us to apply the specified congruence postulate.

1. For ASA $\qquad$
For SAS $\qquad$

2. 

For SAS $\qquad$
For SSS $\qquad$


## Congruent Triangle Proofs

Example 1: Proving Triangles Congruent
GIVEN: $\triangle A B C, \overline{C D} \perp \overline{A B}$
$D$ midpoint of $\overline{A B}$.

PROVE: $\triangle A C D \cong \triangle B C D$


| STATEMENTS | REASONS |
| :---: | :---: |
| 1) $\overline{C D} \perp \overline{A B}$ D midpoint of $\overline{\mathrm{AB}}$. | 1) Given |
| 2) $\overline{A D} \cong \overline{D B} \quad(s \cong s)$ | 2) A midpoint divides a segment into two congruent segments. |
| 3) $\angle A D C$ is a right angle $\angle B D C$ is a right angle. | 3) Perpendicular lines form right angles. |
| 4) $\angle \mathrm{ADC} \cong \angle B D C \quad(\mathrm{a} \cong \mathrm{a})$ | 4) All right angles are congruent. |
| 5) $\overline{\mathrm{CD}} \cong \overline{\mathrm{CD}}$ (s $\mathrm{s}^{\text {s }}$ | 5) Reflexive postulate. |
| 6) $\triangle A C D \cong \triangle B C D$ | 6) s.a.s. $\cong$ ¢ s.a.s. |

## Model Problem \#1

Given: $\overline{A C} \cong \overline{B C}$
$D$ is the midpoint of $\overline{A B}$
Prove: $\triangle A C D \cong \triangle B C D$


Statements
$\square \begin{aligned} & 1 \\ & 2\end{aligned}$
$\square$
3 $\qquad$
$\square 4 \ldots$
$5 \triangle A C D \cong \triangle B C D$
Reasons

2) Given: $\overline{A C}$ and $\overline{D F}$ bisect each other at E

Prove: $\quad \triangle D E A \cong \triangle F E C$

3) Given: $\overline{S R} \perp \overline{R T} ; \overline{T U} \perp \overline{U S} ; ~ \Varangle S T R \cong \Varangle T S U$ Prove: $\triangle T R S \cong \triangle S U T$


LEVEL B
4) Given: $\angle 3 \cong \angle 6$, $\overline{\mathrm{KR}} \cong \overline{\mathrm{PR}}$,
$\angle K R O \cong \angle P R M$
Prove: $\triangle \mathrm{KRM} \cong \triangle \mathrm{PRO}$

5. Given: $\overline{F A} \perp \overline{A D}, \overline{E D} \perp \overline{A D}$, $\overline{A C} \cong \overline{D B}, \Varangle F \cong \Varangle E$

Prove: $\triangle A B F \cong \triangle D C E$.


Example 1: Proving Triangles Congruent

GIVEN: $\triangle A B C, \quad \overline{C D} \perp \overline{A B}$ D midpoint of $\overline{\mathrm{AB}}$.

PROVE: $\triangle A C D \cong \triangle B C D$


| STATEMENTS | REASONS |
| :---: | :---: |
| 1) $\overline{C D} \perp \overline{A B}$ <br> D midpoint of $\overline{\mathrm{AB}}$. | 1) Given |
| 2) $\overline{A D} \cong \overline{D B} \quad(s \cong s)$ | 2) A midpoint divides a segment into two congruent segments. |
| 3) $\angle A D C$ is a right angle $\angle B D C$ is a right angle. | 3) Perpendicular lines form right angles. |
| 4) $\angle \mathrm{ADC} \cong \angle \mathrm{BDC}$ ( $\mathrm{a} \cong$ a) | 4) All right angles are congruent. |
| 5) $\overline{C D} \cong \overline{C D} \quad(\mathrm{~s} \cong \mathrm{~s})$ | 5) Reflexive postulate. |
| 6) $\triangle A C D \cong \triangle B C D$ | 6) s.a.s. $\cong$ ¢ s.a.s. |

## Exit Ticket

In the diagram below of $\triangle A G E$ and $\triangle O L D$,
$\angle G A E \cong \angle L O D$, and $\overline{A E} \cong \overline{O D}$.


To prove that $\triangle A G E$ and $\triangle O L D$ are congruent by
SAS, what other information is needed?

1) $\overline{G E} \cong \overline{L D}$
2) $\overline{A G} \cong \overline{O L}$
3) $\angle A G E \cong \angle O L D$
4) $\angle A E G \cong \angle O D L$

## Practice with Congruent Triangles

1. Given: $\overline{A E} \perp \overline{E D}$

$$
\overline{B C} \perp \overline{C D}
$$

$D$ is the midpoint of $\overline{E C}$.
$\Varangle 3 \cong \Varangle 4$
Prove: $\triangle A E D \cong \triangle B C D$

2. Given: $\overline{A C} \cong \overline{C B}$
$\overline{C D}$ Bisects $\overline{A B}$
Prove: $\triangle \mathrm{ADC} \cong \triangle B D C$

3.

Given: $\overline{R S}$ bisects $\overline{P Q}$ at $T, \overline{P Q}$ bisects $\overline{R S}$ at $T$.
Prove: $\triangle P T S \cong \triangle Q T R$.

4. Given: $\Varangle B A C \cong \Varangle B C A$
$\overline{C D}$ bisects $\Varangle B C A$
$\overline{A E}$ bisects $\Varangle B A C$
Prove: $\triangle \mathrm{ADC} \cong \triangle$ CEA

5. Given: $\overline{S R}$ and $\overline{S T}$ are straight lines.
$\overline{S X} \cong \overline{S Y}$
$\overline{X R} \cong \overline{Y T}$
Prove: $\Delta$ RSY $\cong \Delta$ TSX

6. Given: $\overline{D A} \cong \overline{C B}$
$\overline{D A} \perp \overline{A B}$
$\overline{C B} \perp \overline{A B}$
Prove: $\triangle \mathrm{DAB} \cong \triangle$ CBA

7. Given: $\overline{L M}$ is a straight line

$$
\overline{C B} \cong \overline{D A}
$$

$$
\Varangle 1 \cong \Varangle 2
$$

Prove: $\triangle A B C \cong \triangle B A D$


## Day 4 - CPCTC

SWBAT: To use triangle congruence and CPCTC to prove that parts of two triangles are congruent.

## Warm-Up

What additional information would you need to prove these triangles congruent by ASA?
$\qquad$


With SSS, SAS, ASA, and AAS, you know how to use three parts of triangles to show that the triangles are congruent. Once you have triangles congruent, you can make conclusions about their other parts because, by definition, corresponding parts of congruent triangles are congruent. You can abbreviate this as CPCTC.

CPCTC Proofs
Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
To use CPCTC, you must first show the triangles are congruent!!!

## Example 2: USING CPCTC

Given: B is the midpoint of $\overline{A C}, \overline{A D} \cong \overline{C D}$
Prove: $\angle D A B \cong \angle D C B$


| Statements | Reasons |
| :--- | :--- |
| (s) 1) B is the midpoint of $\overline{A C}, \overline{A D} \cong \overline{C D}$ | 1) $\underline{\text { Given }}$ |
| (s) 2) $\overline{A B} \cong \overline{C B}$ 2) Midpoint $\rightarrow \mathbf{2} \cong$ segments <br> (s) 3) $\overline{D B} \cong \overline{D B}$ 3) Reflexive property of segments <br> 4) $\triangle A D B \cong \triangle C D B$ 4) $\underline{\mathrm{SSS}}$ <br> 5) $\overline{\angle D A B \cong \angle D C B}$ 5) CPCTC |  |

## You Try It!

Given: C is the midpoint of $\overline{B D}, \overline{A C} \perp \overline{B D}$
Prove: $\angle B A C \cong \angle D A C$


| Statements | Reasons |
| :---: | :--- |
| 1) $\overline{\text { 1) }} \overline{\text { 1) }} \overline{B C} \cong \overline{D C}$ | 2) |
| $\left.\begin{array}{ll}\text { 3) } & \angle A C B \& \angle A C D \text { are right angles } \\ \square \text { 4) } \angle A C B \cong \angle A C D & \text { 3) } \\ \square \text { 5) } & \text { 4) } \\ \hline \text { 6) } \triangle A C B \cong \triangle A C D & \text { 5) Reflexive Property } \\ \text { 7) } & \text { 6) } \\ \hline\end{array}\right]$ |  |

## Example 1:

Given: W is the midpoint of $\overline{X Z}, \overline{X Y} \cong \overline{Z Y}$
Prove: $\angle X Y W \cong \angle Z Y W$


| Statements |  |
| :--- | :--- |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |

Example 2: Given: $\overline{P R}$ bisects $\angle Q P S$ and $\angle Q R S$.
Prove: $\overline{P Q} \cong \overline{P S}$


| Statements |  |
| :--- | :---: |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |

3. Given: $\overline{A F C D}, \overline{E D} \perp \overline{D A}, \overline{B A} \perp \overline{D A}, \overline{D C} \cong \overline{A F}$, and $\angle E \cong \angle B$. Prove: $\overline{E F} \cong \overline{B C}$.

| Statements |  | Reasons |
| :--- | :--- | :--- |
| 1 | 1 |  |
| 2 | 2 |  |
| 3 | 3 |  |
| 4 | 4 |  |
| 5 | 5 |  |
| 6 | 6 |  |
| 7 | 8 |  |


4. Given: In $\triangle A C B, \overline{A C} \cong \overline{B C}$ and $\angle A D B \cong \angle B E A$. Prove: $\overline{A E} \cong \overline{B D}$.

| Statements |  |  |
| :--- | :--- | :---: |
| 1 | Reasons |  |
| 2 | 1 |  |
| 3 | 2 |  |
| 4 | 3 |  |
| 5 | 4 |  |
| 6 | 5 |  |
| 7 | 6 |  |
| 8 | 8 |  |

## SUMMARY

Given: B is the midpoint of $\overline{A C}, \overline{A D} \cong \overline{C D}$
Prove: $\angle D A B \cong \angle D C B$


## Warm - Up

In the diagram below, $\triangle X Y V \cong \triangle T S V$.


Which statement can not be proven?

1) $\angle X V Y \cong \angle T V S$
2) $\angle V Y X \cong \angle V U T$
3) $X Y \cong T S$
4) $\overline{Y V} \cong \overline{S V}$

## C.P.C.T.C. and BEYOND

## Auxiliary Lines

A diagram in a proof sometimes requires lines, rays, or segments that do not appear in the original figure. These additions to diagrams are auxiliary lines.

## Ex 1: Consider the following problem.



This proof would be easy if

## Theorem:

## Ex 2:

Given: G is the midpt. of $\overline{\mathrm{FH}}$.

$$
\overline{\mathrm{EF}} \cong \overline{\mathrm{EH}}
$$

Prove: $\angle 1 \cong \angle 2$


## Ex 3: CPCTC and Beyond

Given: $\overline{A D} \cong \overline{C D}$

$$
\measuredangle A D B \cong \measuredangle C D B
$$

Prove: $\overline{D B}$ is the median to $\overline{A C}$


| Statements | Reasons |  |
| :--- | :--- | :--- |
| 1. $\overline{A D} \cong \overline{C D}$ | 1. Given |  |
| 2. $\measuredangle A D B \cong \measuredangle C D B$ | 2. Given |  |
| 3. | 3. |  |
| 4. $\triangle A B D \cong \triangle C B D$ | 4. |  |
| 5. | 5. CPCTC |  |
| 6. |  | 6. |
| 7. $\overline{D B}$ is the median to $\overline{A C}$ | $\mathbf{7 .}$ |  |

Defn: A circle is the set of all points in a plane that are a given distance from a point located at its center. This distance is called the radius. (plural - radii). A circle consists only of of a "rim" but is named by the point in its center -even though the center is not an element of the circle.

## Theorem: All radii of a circle are congruent!

Given: $\bigcirc$

Prove: $\quad \overline{\mathbf{R S}} \cong \overline{\mathbf{U T}}$


Reasons

Statements

1. $) 0$

S 2. $\overline{\mathrm{RO}} \cong \overline{\mathrm{UO}}$
s 3. $\overline{\mathrm{TO}} \cong \overline{\mathrm{SO}}$
A 4. $\angle R O S \cong \angle U O T$
5. $\triangle R O S \cong \triangle U O T$
6. $\overline{\mathbf{R S}} \cong \overline{\mathbf{U T}}$

1. Given
2. All radii of $\mathbf{a} \odot$ are $\cong$
3. Same as 2
4. Vertical $\angle \mathrm{s}$ are $\cong$ (VAT)
5. SAS (Steps 2, 4, 3)
6. СРСТС

Example 4: Given: $\odot \mathrm{Q}, \overline{R P} \cong \overline{S P}$ Prove: $\overrightarrow{P Q}$ bisects $\Varangle R P S$


Reasons

## Example 5:

Given: $\odot C, \Varangle A B X \cong \Varangle E D Y$
Prove: C is the midpoint of $\overline{B D}$


Given: $\quad \mathbf{G}$ is the midpoint of $\overline{\mathbf{H F}}$ $\overline{E F} \cong \overline{\mathrm{EH}}$

Prove: $\quad \angle 1 \cong \angle 2$


1. Given
2. Defn. of midpoint
3. Given
4. Auxiliary Lines
5. Reflexive Property
6. SSS (Steps 2, 3, 5)
7. СРСТС
8. Linear Pair Thm.
9. Linear Pair Thm.
10. Supps of $\cong \angle \mathrm{s}$ are $\cong$

## Exit Ticket

If $\triangle J K L \cong \triangle M N O$, which statement is always true?

1) $\angle K L J \cong \angle N M O$
2) $\angle K J L \cong \angle M O N$
3) $J L \cong M O$
4) $\overline{J K} \cong \overline{O N}$

## Day 6 - Isosceles Triangle Proofs

Isosceles triangles are common in the real world. You can find them in structures such as bridges and buildings. The congruent sides of an isosceles triangle are its legs. The third side is the base. The two congruent sides form the vertex angle. The other two angles are the base angles.


| Theorem | Examples |
| :---: | :---: |
| Isosceles Triangle Theorem <br> If two sides of a triangle are congruent, then the angles opposite the sides are congruent. <br> (If $\Delta$, then $\triangle \Delta$.) | If $\overline{R T} \cong \overline{R S}$, then $\angle T \cong \angle S$. |
| Converse of Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent. $\text { (If } \triangle \text {, then } \triangle \text {.) }$ | If $\angle N \cong \angle M$, then $\overline{L N} \cong \overline{L M}$. |

Practice Problems:

1. Given: Isosceles triangle $A B C$ with base $\overline{A B}$ $M$ is the midpoint of $A B$
$\overline{A D} \cong \overline{B E}$
Prove: $\overline{D M} \cong \overline{M E}$


| Statements |  |
| :--- | :--- |
| 1 | 1 |
| 2 | Reasons |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |

$$
\text { Given: } \begin{aligned}
\overline{C A} & \cong \overline{C B} \\
\angle P A B & \cong P B A
\end{aligned}
$$

Prove: $\triangle E P A \cong \triangle D P B$


| Statements |  |
| :--- | :--- |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 7 |

If $\overline{C A} \cong \overline{C B}$, and $\overline{D A} \cong \overline{E B}$, prove that $\angle 1 \cong \angle 2$.


| Statements |  |  |
| :--- | :--- | :---: |
| 1 | Reasons |  |
| 2 | 1 |  |
| 3 | 2 |  |
| 4 | 3 |  |
| 5 | 4 |  |
| 6 | 5 |  |
| 7 | 6 |  |
| 8 | 7 |  |

Given $\overline{A D} \cong \overline{B E}, \overline{C D} \cong \overline{C E}$, and $\overline{A D E B}$, prove that $\overline{A C} \cong \overline{B C}$.


| Statements |  |
| :--- | :--- |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 7 |

Given: $\overline{D G} \cong \overline{C G}$,
$\overline{A D} \cong \overline{F C}$
$\overline{B C} \cong \overline{E D}$

Prove: $\angle \mathrm{B} \cong \angle \mathrm{E}$

| Statements |  | Reasons |  |
| :--- | :--- | :---: | :---: |
| 1 | 1 |  |  |
| 2 | 2 |  |  |
| 3 | 3 |  |  |
| 4 | 4 |  |  |
| 5 | 5 |  |  |
| 6 | 6 |  |  |
| 7 | 7 | 30 |  |

Summary of Isosceles Triangles

Given: Isosceles triangle $A B C$ with bose $\overline{A B}$ $M$ is the midpoint of $A B$ $\overline{A D} \cong \overline{B E}$

Prove: $\overline{D M} \cong \overline{M E}$
Plan: $\triangle A D M \cong \triangle B E M$ by $S A S$

Statements
1 Isosceles triangle ABC with bose $\overline{A B}$
$2^{M}$ is the midpoint of $A B$
S $3 \overline{A D} \cong \overline{B E}$
${ }^{4} \overline{C A} \cong \overline{C B}$
$A 5 \Varangle A \cong \Varangle B$
$S_{6} \overline{A M} \cong \overline{B M}$
${ }_{7} \triangle A D M \cong \triangle B E M$
$8 \overline{D M} \cong \overline{M E}$


Reasons
$\left.\begin{array}{l}1 \\ 2 \\ 3\end{array}\right\}$ Given
4 Isosceles $\Delta \rightarrow 2$ legs $\cong$
5 If $\Delta$, then $\Delta$
6 Midpoint $\rightarrow 2 \cong$ segments
$7 \operatorname{SAS}(3,5,6)$
8 CPCTC

## Exit Ticket

Use the figure for Exercises 1 and 2.


1. What postulate or theorem proves
$\overline{H G} \cong \overline{F G}$ ?
A Isosceles Triangle Theorem (If $\Delta$, then $\triangle \Delta$.)
B Converse of Isosceles Triangle (If $\Delta$, then $\ell$.) Theorem
2. If $\triangle F G J \cong \triangle H G J$, what reason justifies the statement $\angle H G J \cong \angle F G J$ ?
A ASA
B Reflex. Prop. of $\cong$
C Def. of bisects
D CPCTC

## Day 7-Hy-Leg

Warm - Up
Given: $\angle \mathrm{BDE} \cong \angle \mathrm{BED}$
$\angle A B E \cong \angle C B D$

Prove: $\triangle A B C$ is Isosceles


| Statements | Reasons |
| :---: | :---: |

If the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other, then the two right triangles are congruent. (HL)

| Hypotenuse-Leg (HL) Congruence Theorem |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of |  |  |  |  |  |  |
| another right triangle, then the triangles are congruent. |  |  |  |  |  |  |

To use the HL Theorem, you must show that these three conditions are met:

- There are two right triangles
- There is one pair of $\cong$ hypotenuses
- There is one pair of $\cong$ legs


## What additional information would you need to prove the triangles congruent by the HL Theorem?


2.

3.


Given: $\overline{O F}$ is an altitude in Circle 0. Prove: $\overline{\boldsymbol{E F}} \cong \overline{\boldsymbol{F G}}$


| Statements |  |
| :--- | :--- |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 8 |

2. Given: $\measuredangle P, \measuredangle R$ are right angles $\overline{P S} \cong \overline{Q R}$
Prove: $\quad \triangle P Q S \cong \triangle R S Q$ Statements

[^0]1

2
3
4
5
3. Given: $\overline{A D}$ is the $\perp$ bisector of $\overline{\mathrm{EC}}$

Prove: | $\overline{C D} \cong \overline{E A}$ |
| :--- |
| $\triangle C D B \cong \triangle E B A$ |



| Statements |  |  |
| :--- | :--- | :---: |
| 1 | Reasons |  |
| 2 | 1 |  |
| 3 | 2 |  |
| 4 | 3 |  |
| 5 | 4 |  |
| 6 | 5 |  |
| 7 | 6 |  |
| 8 | 7 |  |

4. Given: $\overline{\mathrm{AE}} \cong \overline{\mathrm{CF}}$, $\overline{\mathrm{AB}} \cong \overline{\mathrm{CD}}$;
$\angle B F A$ is a right angle. $\angle D E C$ is a right angle. Prove: $\triangle \mathrm{CDE} \cong \triangle \mathrm{ABF}$


| Statements |  |  |
| :--- | :--- | :---: |
| 1 | Reasons |  |
| 2 | 1 |  |
| 3 | 2 |  |
| 4 | 3 |  |
| 5 | 4 |  |
| 6 | 5 |  |
| 7 | 6 |  |
| 8 | 7 |  |

Given: $\overline{\mathrm{BC}} \perp \overline{\mathrm{AC}}$
$\overline{B D} \perp \overline{A D}$
$\overline{\mathbf{A C}} \cong \overline{\mathbf{A D}}$
Prove: $\quad \overrightarrow{A B}$ bisects $\angle C A D$

Statements

1. $\overline{B C} \perp \overline{A C}$
$R$ 2. $\angle A C B$ is a right $\angle$
2. $\overline{B D} \perp \overline{A D}$
$R$ 4. $\angle A D B$ is a right $\angle$
L 5. $\overline{\mathrm{AC}} \cong \overline{\mathrm{AD}}$
H 6. $\overline{\mathbf{A B}} \cong \overline{\mathrm{AB}}$
3. $\triangle A C B \cong \triangle A D B$
4. $\angle C A B \cong \angle D A B$
5. $\overrightarrow{A B}$ bisects $\angle C A D$


Reasons

1. Given
2. Definition of $\perp$ Segments
3. Given
4. Same as 2
5. Given
6. Reflexive Property
7. $\mathrm{HL}(2,4,6,5)$
8. СРСТС
9. Definition of $\angle$ Bisector

## Exit Ticket

For these triangles, select the triangle congruence statement and the postulate or theorem that supports it.



1) $\triangle A B C \cong \triangle J L K, H L$
2) $\triangle A B C \cong \triangle J K L, \mathrm{HL}$
3) $\triangle A B C \cong \triangle J L K$, SAS
4) $\triangle A B C \cong \triangle J K L, S A S$

## Day 8 -

## Right Angle Theorems \& Equidistance Theorem

Theorem: If two angles are both supplementary and congruent, then they are right angles.
(Ł's $\cong \& S u p p l . \rightarrow$ right angles)

Given: $\angle 1 \cong \angle 2$


Conclusion: $\qquad$
*** Proving that lines are perpendicular depends on you proving that they form $\qquad$ .

1. Given: $\odot P$

$$
\text { Prove: } \quad \frac{S}{P S} \perp \overline{Q R} \text { ithe midpoint of } \overline{Q R} .
$$



| Statements |  |
| :--- | :--- |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 6 |
| 7 | 8 |
| 8 | 8 |

## EQUIDISTANCE THEOREM

Definition: The distance between two objects is the length of the shortest path joining them.

Postulate: A line segment is the shortest path between two points.
If two points $P$ and $Q$ are the same distance from a third point, $X$, they are said to be equidistant from X .

## Picture:

| Statement | Means..... |
| :--- | :--- |
| 1. $\overline{A C} \cong \overline{B C}$ |  |
|  |  |
| 2. $\overline{M Q} \cong \overline{N Q}$ |  |
| 3. $\overline{D F} \cong \overline{G F}$, and |  |
| $\frac{B F}{} \cong \overline{E F}$ |  |
|  |  |


(Please highlight segment CD and put a circle around points $A$ and $B$.)
These diagrams have something in common. In each, both points $A$ and $B$ are equidistant from the endpoints $\qquad$ and ____ of segment $\qquad$ . You can prove that line $A B$ is the perpendicular bisector of segment $C D$.

Definition: The perpendicular bisector of a segment is the line that bisects and is perpendicular to the segment.
[-- Equidistance Theorem -
If if two points are each equidistant from the endpoints of a segment, then , the two points determine the perpendicular bisector of that segment.

Given: $\overline{P A} \cong \overline{P B}, \overline{Q A} \cong \overline{Q B}$

Conclusion: $\qquad$


B
2. Given: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

Prove: $\overrightarrow{A E} \perp$ bis. $\overline{B D}$


Statements
Reasons

| Statements |  |
| :--- | :--- |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

3. Given: $\odot o$

Prove: $\begin{aligned} & \frac{\Delta A B C \text { isosceles, }}{\overline{A D}} \perp \text { bis. } \overline{B C}\end{aligned}$


| Statements |  |
| :--- | :--- |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 7 |

## WHY the Equidistance Theorem?

Given: $\overline{A B} \cong \overline{A D}$

$$
\overline{B C} \cong \overline{C D}
$$

Prove: $\overline{A C}$ is the $\perp$ bisector of $\overline{B D}$

## Statements

1. $\overline{A B} \cong \overline{A D}, \overline{B C} \cong \overline{D C}$
2. $\overline{A C} \cong \overline{A C}$
3. ${ }^{\triangle} \mathrm{ABC} \cong \triangle{ }^{\triangle} \mathrm{ADC}$
4. $\angle \mathrm{BAC} \cong \angle \mathrm{DAC}$
5. $\overline{A E} \cong \overline{A E}$
6. ${ }^{\triangle} \mathrm{BAE} \cong{ }^{\triangle} \mathrm{DAE}$
7. $\angle \mathrm{AEB} \cong \angle \mathrm{AED}$
8. $\angle \mathrm{AEB}$ is suppl. to $\angle \mathrm{AED}$
9. $\angle \mathrm{AEB}$ and $\angle \mathrm{AED}$ are right angles
10. $\overline{A C} \perp \overline{B D}$
11. $\overline{B E} \cong \overline{D E}$
12. E is the midpoint of $\overline{B D}$
13. $\overline{A C}$ is the $\perp$ bisector of $\overline{B D}$

Reasons

1. Given
2. Reflexive
3. SSS

4. CPCTC
5. Reflexive
6. SAS
7. CPCTC
8. L.P.'s form suppl. $\angle \mathrm{s}$
9. $九^{\prime}$ § \& Suppl. $\rightarrow$ right angles
10. right $\Varangle s \rightarrow \perp$
11. CPCTC
12. Definition of Midpoint
13. Definition of $a \perp$ bisector

Given: $\overline{A B} \cong \overline{A D}$
$\overline{B C} \cong \overline{C D}$
Prove: $\overline{A C}$ is the $\perp$ bisector of $\overline{B D}$


Statements
Reasons
1 $\overline{A B} \cong \overline{A D}$
$2 \overline{\boldsymbol{B C}} \cong \overline{\boldsymbol{C D}}$
$3 \overline{A C}$ is the $\perp$ bisector of $\overline{B D}$
) Given
Given
${ }_{3}$ Equidistance Thm(1, 2)

Given: $\quad \overrightarrow{P Q}$ is the $\perp$ bisector of $\overrightarrow{A B}$

Conclusion: $\qquad$

4.

Given: $\overline{A D} \perp$ bis. $\overline{\boldsymbol{B C}}$
Prove: $\Varangle 1 \cong \Varangle 2$


| Statements |  |
| :--- | :---: |
| 1 | Reasons |
| 2 | 1 |
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |

## SUMMARY



If you know that $\overline{X W} \cong \overline{X Y}$ and $\overline{Z W} \cong \overline{Z Y}$,
then you can conclude that $\overparen{X Z}$ is the perpendicular bisector of $\overline{W Y}$.

Statements


Reasons

1. $\overline{\mathrm{AB}}=\overline{\mathrm{AD}}$
2. $\overline{B C}=\overline{C D}$
3. $\overline{\mathrm{AE}}$ is the $\perp$ bisector of $\overline{\mathrm{BD}}$
4. Given
5. Given
6. ET $(1,2)$ (If two points are each equidistant from the endpoints of a segment, the the two points determine the perpendicular bisector of that segment).
7. Converse of ET (If a point is on the $\perp$ bisector of a segment, then it is equidistant from the endpoints of the segment).

## Exit Ticket

Given: $\stackrel{\leftrightarrow}{W}$ is the perpendicular bisector of $\overline{X Z}$. Which statement is true?

A. $Y$ is the midpoint of $\bar{W}$.
B. $\overline{X Y} \cong \overline{Y V}$
C. $\angle Y V Z$ is a right angle.
D. $\overline{X Y} \cong \overline{Y Z}$

## Day 9 - Detour Proofs

## Warm - Up

Given: $\odot O$
Prove: $\quad \begin{aligned} & \Varangle Y X Z \cong \measuredangle Y W Z \\ & \overline{O Y} \perp \text { bis. } \overline{X W}\end{aligned}$


Statements
Reasons

| 1 | 1 |
| :--- | :--- |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |

Sometimes, it is impossible to use the given in order to prove immediately that a particular pair of triangles is congruent. In such cases, the given may contain enough information to first prove another pair of triangles congruent. Then, corresponding congruent parts in these congruent triangles may be used to prove the original pair of triangles congruent. See how this is done in the following example.

Example 1:
Given: $\overline{A E B}, \overline{A C} \cong \overline{A D}$, and $\overline{C B} \cong \overline{D B}$
Prove: $\triangle A C E \cong \triangle A D E$


Whenever you are asked to prove that triangles or parts of triangles are congruent and you suspect a detour may be needed, use the following procedures.

## I Procedure for Detour Proofs

1. Determine which triangles you must prove congruent to reach the desired conclusion
2. Attempt to prove those triangles congruent - if you cannot due to a lack of information - it's time to take a detour...
3. Find a different pair of triangles congruent based on the given information
4. Get something congruent by CPCTC
5. Use the CPCTC step to now prove the triangles you wanted congruent.

Example 2:
Given: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$
Prove: $\triangle C D E \cong \triangle C B E$


| Statements | Reasons |
| :--- | :--- |
| $1 \npreceq 1 \cong \npreceq 2, \Varangle 3 \cong \not 44$ | 1 Given |
| 2 | 2 Reflexive Property |
| $3 \triangle C B A \cong \triangle C D A$ | 3 |
| 4 | 4 CPCTC |
| 5 | 5 Reflexive Property |
| $6 \triangle C D E \cong \triangle C B E$ | 6 |

Example 3:
Given: $\triangle \mathrm{ABC} \cong \triangle \mathrm{EBC}$
$\overline{\mathrm{DB}}$ is a median to $\overline{\mathrm{AE}}$
Prove: $\triangle A C D \cong \triangle E C D$


Statements
Reasons

| Statements |  | Reasons |
| :--- | :--- | :--- |
| 1 | 1 |  |
| 2 | 2 |  |
| 3 | 3 |  |
| 4 | 4 |  |
| 5 | 5 |  |
| 6 |  | 6 |
| 7 | 8 |  |

## Example 4:

Given: $\overline{P Q}$ bisects $\overline{Y Z}$
Q is the midpoint of $\overline{\mathrm{WX}}$

$$
\begin{aligned}
& \angle \mathrm{Y} \cong \angle \mathrm{Z} \\
& \overline{\mathrm{WZ}} \cong \overline{\mathrm{XY}}
\end{aligned}
$$

Prove: $\angle \mathrm{WQP} \cong \angle \mathrm{XQP}$


| Statements | Reasons |
| :--- | :--- |

## SUMMARY

Given: $\overline{P Q}$ bisects $\overline{Y Z}$.
$Q$ is the midpt. of $\overline{W X}$.
$\angle Y \cong \angle Z, \overline{W Z} \cong \overline{X Y}$
Prove: $\angle W Q P \cong \angle X Q P$


Reasons

1. Given
2. Def. of segment bis.
3. Def. of midpt.
4. Given
5. Given
6. SAS $(3,4,5)$
7. CPCTC
8. Given
9. Def. of midpt.
10. Reflexive Property
11. SSS $(7,9,10)$
12. CPCTC

## Exit Ticket

Complete the proof.
Given: $\overline{B C} \cong \overline{D A}, \angle 1 \cong \angle 2$, and $\overline{C F} \cong \overline{A F}$.
Prove: $\triangle C E F \cong \triangle A E F$.

$\angle B E C \cong \angle D E A$ by vertical angles. $\triangle B E C \cong \triangle D E A$ by (a)__. Then by CPCTC, $\overline{C E}$ $\cong \overline{A E} \cdot \overline{E F} \cong \overline{E F}$ by the Reflexive Property. So $\triangle C E F \cong \triangle A E F$ by (b)
A. a. SAS; b. SAS
B. a. AAS; b. SSS
C. a. ASA; b. SSS
D. a. AAS; b. HL

## Day 10 - Missing Diagram Proofs

## Warm - Up

Given: $\underset{\overrightarrow{\mathrm{AB}}}{ } \cong \overline{\mathrm{AC}}$; BD bisects $\angle \mathrm{ABE}$. $\overrightarrow{\mathrm{CD}}$ bisects $\angle \mathrm{ACE}$.
Conclusion: $\overline{\mathrm{AE}}$ bisects $\overline{\mathrm{BC}}$.


Many proofs we encounter will not always be accompanied by a diagram or any given information. It is up to us to find the important information, set up the problem, and draw the diagram all by ourselves!!!


Example 1: If two altitudes of a triangle are congruent, then the triangle is isosceles.

Given:

Prove:

## Example 2: The medians of a triangle are congruent if the triangle is equilateral.

Given:

Prove:

## Example 3: the altitude to the base of an isosceles triangle bisects the vertex angle.

Given:

Prove:

## SUMMARY

Example: Prove that the bisector of the vertex angle of an isosceles triangle is also the median to the base.
Given: $\triangle A B C$ isosceles with base $\overline{B C}$.
$A D$ bisects $\angle B A C$


Prove: $\overline{\mathrm{AD}}$ is a median to base $\overline{\mathrm{BC}}$.

| Statements | Reasons |
| :---: | :---: |
| 1 $\triangle A B C$ is isosceles with base $\overline{B C}$. | 1 Given |
| $2 \overline{B A} \cong \overline{C A}$ | 2 Def. of Isosceles $\Delta$ |
| 3 AD bisects $\angle \mathrm{BAC}$ | 3 Given |
| $4 \angle B A D \cong \angle C A D$ | ${ }^{4}$ Def. of $\angle$ bisector |
| $5 \overline{A D} \cong \overline{A D}$ | 5 Reflexive Prop |
| ${ }_{6} \triangle B A D \cong \triangle C A D$ | ${ }_{6}$ SAS (2, 4, 5) |
| $7 \overline{\mathrm{BD}} \cong \overline{C D}$ | 7 CPCTC |
| ${ }^{8} \mathrm{D}$ is the midnoint of $\overline{\mathrm{BC}}$. | 8 Def of midpoint |
| $9 \overline{\mathrm{AD}}$ is a medianto base $\overline{\mathrm{BC}}$. | 9. Def'n of a median |

## Exit Ticket

In $\triangle B A T$ and $\triangle C R E, \angle A \cong \angle R$ and $\overline{B A} \cong \overline{C R}$. Write one additional statement that could be used to prove that the two triangles are congruent. State the method that would be used to prove that the triangles are congruent.

## ANSWER KEYS

19. Think: Use Ext. $\angle$ Thm.

$$
\begin{gathered}
\mathrm{m} \angle W+\mathrm{m} \angle X=\mathrm{m} \angle X Y Z \\
5 x+2+8 x+4=15 x-18 \\
13 x+6=15 x-18 \\
24=2 x \\
x=12 \\
\mathrm{~m} \angle X Y Z=15 x-18 \\
=15(12)-18=162^{\circ}
\end{gathered}
$$

20. Think: Use Ext. $\angle$ Thm and subst. $\mathrm{m} \angle C=\mathrm{m} \angle D$.
$\mathrm{m} \angle C+\mathrm{m} \angle D=\mathrm{m} \angle A B D$

$$
2 \mathrm{~m} \angle D=\mathrm{m} \angle A B D
$$

$$
2(6 x-5)=11 x+1
$$

$$
12 x-10=11 x+1
$$

$\mathrm{m} \angle C=\mathrm{m} \angle D$

$$
x=11
$$

$$
\begin{aligned}
& =6 x-5 \\
& =6(11)-5=61^{\circ}
\end{aligned}
$$

21. Think: Use Third $\S$ Thm.

$$
\begin{aligned}
\angle N & \cong \angle P \\
\mathrm{~m} \angle N & =\mathrm{m} \angle P \\
3 y^{2} & =12 y^{2}-144 \\
-9 y^{2} & =-144 \\
y^{2} & =16 \\
\mathrm{~m} \angle N & =3 y^{2}=3(16)=48^{\circ} \\
\mathrm{m} \angle P & =\mathrm{m} \angle N=48^{\circ}
\end{aligned}
$$

22. Think: Use Third $\S$ Thm.

$$
\begin{aligned}
\angle Q & \cong \angle S \\
\mathrm{~m} \angle Q & =\mathrm{m} \angle S \\
2 x^{2} & =3 x^{2}-64 \\
64 & =x^{2} \\
\mathrm{~m} \angle Q & =2 x^{2}=2(64)=128^{\circ} \\
\mathrm{m} \angle S & =\mathrm{m} \angle Q=128^{\circ}
\end{aligned}
$$

49. Let $\mathrm{m} \angle A=x^{\circ}$.

$$
\begin{aligned}
& \mathrm{m} \angle B=1 \frac{1}{2}(x)-5 \\
& \mathrm{~m} \angle C=2 \frac{1}{2}(x)-5 \\
& \mathrm{~m} \angle A+\mathrm{m} \angle B+\mathrm{m} \angle C=180 \\
& x+1 \frac{1}{2}(x)-5+2 \frac{1}{2}(x)-5=180 \\
& 5 x-10=180 \\
& 5 x=190 \\
& x=38
\end{aligned}
$$

$$
\mathrm{m} \angle A=x^{\circ}=38^{\circ}
$$

## Day 2 Answers

State if the two triangles are congruent. If they are, state how you know.

3)

5)


Not congruent
7)

9)


SAS
2)

4)

6)

8)


SSS
10)


SSS

State what additional information is required in order to know that the triangles are congruent for the reason given.
11) ASA

$\angle S U T \cong \angle D U T$
13) SAS

15) SAS

17) SSS


$$
\overline{R S} \cong \overline{D Q}
$$


14) ASA

16) ASA

$\angle L \cong \angle T$
18) SAS


Day 3 - Answers

Practice with Congruent Triangles

1. Given: $\overline{A E} \perp \overline{E D}$
$\overline{B C} \perp \overline{C D}$
$D$ is the midpoint of $\overline{E C}$.
$43 \cong 44$
Prove: $\triangle A E D \cong \triangle B C D$

2. Given: $\overline{A C} \cong \overline{C B}$
$\overline{C D}$ Bisects $\overline{A B}$
Prove: $\triangle A D C \cong \triangle B D C$


(2) $x$ and $\Varangle 2$ are right $\Varangle ' J$
(3) $41 \cong 42$ (A)
(4) $D$ is the mapt of $\overline{E C}$
(5) $E D \cong \overline{C D}(S)$
(4) given
(5) def. of mapt
(6) $43 \cong \Varangle 4(A)$
6) given
(7) $\triangle A E D \approx \triangle B C D$
(7) $A S A(3,5,6)$

Statements Reasons
$\begin{array}{ll}\text { (1) } \overline{A C}=\overline{=} \bar{B}(S) & \text { (1) given } \\ \text { (2) } \overline{C D} \text { biects } \overline{A B} & \text { (2) given }\end{array}$
(3) Dis the mapt of $\overline{A B}$ (3) def. of kg biectar
(4) $\overline{A D} \cong \overline{B D}(s)$
(5) $\overline{C D} \cong \overline{C D}(s)$
(6) $\triangle A D C \cong \triangle B D C$
(4) def. of indpt
(5) Reflexive pry
(6)SSS $(1,4,5)$


Prove: $\triangle P T S \cong \triangle Q T R$

(1) $\overline{R S}$ bixcts $\overline{P Q}$ at $T$
(2) $T$ is the mdpt of $\bar{Q}$
(3) $\overline{P T} \cong \overline{Q T}$ (J)
(4) $\overline{P Q}$ biects $\bar{R}$ at $T 4$ given
(5) Tis the mdptot $\bar{R}$ (5) def. of eg bircter
(6) $\overline{R T} \cong \overline{S T}(S)$
(1) $\Varangle 1 \cong \Varangle 2(A)$
(8) $\triangle P T S \cong \triangle Q T R$ (8)SAS $(3,7,6)$
(2) def. of regbixctor
(3) det. of Mdpt
(6) det. of indjut
(1) yertical $x^{\prime \prime}$ are $\cong$
4. Given: $\Varangle B A C \cong \Varangle B C A$
$\overline{C D}$ bisects $\triangle B C A$
$\overline{A E}$ bisects $4 B A C$
Prove: $\triangle A D C \cong \triangle C E A$

(3) $41 \cong 42(A)$
(3) halves of $\cong$ are $\cong$ or

(5) $\triangle A D C \cong \triangle C E A$
(5) $A S A(1,4,3)$
5. Given: $\overline{S R}$ and $\overline{S T}$ are straight lines.
$\frac{\overline{S X}}{\overline{X P}} \cong \overline{\bar{S}} \overline{\bar{Y}}$

$$
\overline{X X} \cong \frac{\Delta Y}{\bar{X}}
$$

Prove: $\triangle$ RS $\cong \triangle T S X$


(2) $\Varangle S \cong \Varangle S(A)$
(3) $\overline{S R} \cong \overline{S T}(S)$
(4) $\triangle R S Y \cong \Delta T S X$
(1) given
(2) Reflexive Prop
(3) Addition Rap. ( 1,1 )
(4) SAS $(1,2,3)$
6. Given: $\overline{D A} \cong \overline{C B}$
$\overline{D A} \perp \overline{A B}$
$\overline{C B} \perp \overline{A B}$
Prove: $\triangle \mathrm{DAB} \cong \triangle C B A$


7. Given: $\overline{L M}$ is a straight line $\overline{C B} \cong \overline{D A}$
Prove: $\triangle A B C=\triangle B A D$



| (2) suppl. to 43  <br>  2 suppl. to $\Varangle 4$ (2) unear Pair thm <br> (3) $\Varangle 3 \cong 44 \quad(A)$ (3) $\cong$ suppl. thm  <br> (4) $\overline{A B} \cong \overline{A B}(S)$ (4) Reflexive Prop.  <br> (5) $\triangle A B C \cong \triangle B A D$ (5) SAS $(1,3,4)$  |
| :--- | :--- |

Answers to Day 4
6. Given: $\odot O, \overline{C D} \cong \overline{D E}$

Prove: $\Varangle C O D \cong \Varangle D O E$


| Statements |  | Reasons |
| :--- | :--- | :--- |
| 1. $\odot O, \overline{C D} \cong \overline{D E}$ | (S) | 1. Given |
| 2. $\overline{C O} \cong \overline{E O}$ | (S) | 2. All radii of a $\odot$ are $\cong$ |
| 3. $\overline{D O} \cong \overline{D O}$ | (S) | 3. Reflexive Property |
| 4. $\triangle C O D \cong \triangle E O D$ | 4. SSS $(1,2,3)$ |  |
| 5. $\Varangle C O D \cong \Varangle D O E$ | 5. CPCTC |  |

12 Given: H is the midpt. of $\overline{\mathrm{GJ}}$. M is the midpt. of $\overline{\mathrm{OK}}$.

$$
\overline{\mathrm{GO}} \cong \overline{\mathrm{JK}}
$$

$$
\overline{\mathrm{GJ}} \equiv \overline{\mathrm{OK}}
$$

$$
\angle G \cong \angle K
$$

$$
\mathrm{OK}=27
$$


$\mathrm{m} \angle \mathrm{GOH}=\mathrm{x}+24, \mathrm{~m} \angle \mathrm{GHO}=2 y-7$,

$$
\mathrm{m} \angle \mathrm{JMK}=3 y-23, \mathrm{~m} \angle \mathrm{MJK}=4 x-105
$$

Find: $\mathrm{m} \angle \mathrm{GOH}, \mathrm{m} \angle \mathrm{GHO}$, and GH

$$
\begin{aligned}
m \times 60 H & =m 4 K J M \\
x+24 & =4 x-105 \\
24 & =3 x-105 \\
129 & =3 x \\
43 & =x \\
m+60 H & =43+24=67^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
M \times G H O & =m 4 K M 1 \\
2 y-7 & =3 y-23 \\
16 & =y \\
m+6 H O & =2(16)-7=25^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
& G H=1 / 26 \\
& 6 H=1 / 2(27)=13.5
\end{aligned}
$$

13 Given: $\angle \mathrm{A} \cong \angle \mathrm{E}$, $\overline{\mathrm{AB}} \cong \overline{\mathrm{BE}}$, $\overline{\mathrm{FB}} \perp \overline{\mathrm{AE}}$,
$\angle 2 \cong \angle 3$
Prove: $\overline{\mathrm{CB}} \cong \overline{\mathrm{DB}}$


| statements | Reasons |
| :---: | :---: |
| ${ }^{1} \angle A \cong \angle E, ~(A)$ | 1. Given |
| $\overline{\mathrm{AB}} \cong \overline{\mathrm{BE}}, \quad(\mathrm{S})$ |  |
| $\overline{\mathrm{FB}} \perp \overline{\mathrm{AE}}$, |  |
| $\angle 2 \cong \angle 3$ |  |
| 2. $\measuredangle F B A$ and $\measuredangle$ FBE arert $\measuredangle s$ | 2. $\perp$ lines $\rightarrow$ right $\Varangle$ |
| 3. $\Varangle 1$ compl. $\ddagger 2$ | 3. If 2 ¢ form $\mathrm{art} \measuredangle, \rightarrow$ |
| Ł3 compl. $\ddagger 4$ |  |
| 4. $\Varangle 1 \cong \Varangle 4$ (A) | 4. Congruent Compl. Thm |
| 5. $\triangle A C B \cong \triangle E D B$ | 5. ASA ( $1,1,4$ ) |
| 6. $\overline{C B} \cong \overline{D B}$ | 6. CPCTC |

18 Given: $\overline{\mathrm{KG}} \cong \overline{\mathrm{GJ}}$,
$\angle 2 \cong \angle 4$,
$\angle 1$ is comp. to $\angle 2$.
$\angle 3$ is comp. to $\angle 4$.
$\angle \mathrm{FGJ} \cong \angle \mathrm{HGK}$
Conclusion: $\overline{\mathrm{FG}} \cong \overline{\mathrm{HG}}$


Statements
Reasons

| 1. $\overline{\mathrm{KG}} \cong \overline{\mathrm{GJ}}, \mathrm{S})$ | 1. Given |
| :--- | :--- |
| $\angle 2 \cong \angle 4$, |  |
| $\angle 1$ is comp. to $\angle 2$. |  |
| $\angle 3$ is comp. to $\angle 4$. | 2. Congruent Compl. Thm |
| $\angle \mathrm{FGJ} \cong \angle \mathrm{HGK}$ | 3. Reflexive Property |
| 2. $\Varangle 1 \cong \Varangle 3$ (A) | 4. Subtraction Postulate (1,3) |
| 3. $\Varangle K G J \cong \Varangle K G J$ | 5. ASA $(2,1,4)$ |
| 4. $\Varangle F G K \cong \Varangle H G J$ (A) | 6. CPCTC |
| 5. $\Delta F G K \cong \triangle H G J$ |  |

21 Given: $\overline{\mathrm{AE}} \cong \overline{\mathrm{FC}}$, $\overline{\mathrm{FB}} \cong \overline{\mathrm{DE}}$, $\angle \mathrm{CFB} \cong \angle \mathrm{AED}$
Prove: $\angle 1 \cong \angle 2$


Statements

1. $\overline{\mathrm{AE}} \cong \overline{\mathrm{FC}}$, $\overline{\mathrm{FB}} \cong \overline{\mathrm{DE}}, \quad(\mathrm{S})$ $\angle C F B \cong \angle A E D$
2. $\overline{E F} \cong \overline{E F}$
3. $\overline{A F} \cong \overline{C E}$ (S)
4. $\Varangle C F B$ Suppl. $\Varangle B F A$ $\Varangle A E D$ Suppl. $\Varangle D E C$
5. $\Varangle B F A \cong \measuredangle D E C \quad(\mathrm{~A})$
6. $\triangle B F A \cong \triangle D E C$
7. $\Varangle 1 \cong \Varangle 2$

Reasons

1. Given
2. Reflexive Property
3. Addition Postulate (1, 2)
4. Linear Pair Thm
5. Congruent Suppl. Thm
6. $\operatorname{SAS}(1,5,3)$
7. CPCTC

## Answers to Day 5



1 a Median b Altitude c Altitude d Both

2 Given: $\overline{\mathrm{HJ}} \cong \overline{\mathrm{KJ}}$
$\angle \mathrm{MJH} \cong \angle \mathrm{MJK}$
Prove: $\overrightarrow{\mathrm{MJ}}$ bis $\angle \mathrm{HMK}$.
1 Given
2 Given
3 Reflexive prop
4 SAS
5 CPCTC
$2 \cong \angle \mathrm{~s}$, it bis the $\angle$.

3 Given: $\overline{\mathrm{NR}} \cong \overline{\mathrm{PR}}$
$\overrightarrow{\mathrm{RO}}$ bis $\angle \mathrm{NRP}$.
Prove: $\quad \overrightarrow{\mathrm{OR}}$ bis $\angle$ NOP.
$1 \overline{\mathrm{NR}} \cong \overline{\mathrm{PR}}$
$2 \overrightarrow{\mathrm{RO}}$ bis $\angle \mathrm{NRP}$
$3 \angle \mathrm{NRO} \cong \angle \mathrm{PRO}$
$4 \overline{\mathrm{OR}} \cong \overline{\mathrm{OR}}$
4 Reflexive prop
5 SAS
CPCTC $2 \cong \angle \mathrm{~s}$, it bis the $\angle$


1 Given
2 Given
An alt of a $\Delta$ forms rt $\angle \mathrm{s}$ is drawn.
$4 \mathrm{Rt} \angle \mathrm{s}$ are $\cong$.
5 Reflexive prop
6 ASA
7 CPCTC
8 If a seg from the vertex of into $2 \cong$ segs, it is the median.

5 Given: $\odot 0$

$$
\overline{\mathrm{GJ}} \cong \overline{\mathrm{HJ}}
$$

Prove: $\angle \mathrm{G} \cong \angle \mathrm{H}$
$1 \odot 0$
$2 \overline{\mathrm{OG}} \cong \overline{\mathrm{OH}}$
$3 \overline{\mathrm{GJ}} \cong \overline{\mathrm{HJ}}$
4 Draw $\overline{\mathrm{OJ}}$
$5 \overline{\mathrm{OJ}} \cong \overline{\mathrm{OJ}}$
1 Given


2 Radii of a $\odot$ are $\cong$.
3 Given
4 Two pts determine a seg.
$6 \triangle \mathrm{OGJ} \cong \triangle \mathrm{OHJ}$
5 Reflexive prop
$7 \angle \mathrm{G} \cong \angle \mathrm{H}$
6 SSS
7 CPCTC
$6 \quad \overline{\mathrm{SW}} \cong \overline{\mathrm{VW}}$ by def of a median so, $2 x+30=5 x-6$

$$
36=3 x
$$

$$
12=x
$$


$\overline{\mathrm{SW}}=2(12)+30$
$\overline{\mathrm{SW}}=24+30$
$\overline{\mathrm{WV}}=60-6 \quad \overline{\mathrm{ST}}=52$
$\overline{\mathrm{SW}}=54$
$\overline{\mathrm{WV}}=54$

7 Given: $\overline{\mathrm{KP}}$ is a median.
$\overline{\mathrm{MK}} \cong \overline{\mathrm{RK}}$
Concl: $\angle 3 \cong \angle 4$
$1 \overline{\mathrm{KP}}$ is a median.
$2 \overline{\mathrm{RP}} \cong \overline{\mathrm{MP}}$
1 Given
2 A median of a $\Delta$ divides one side into $2 \cong$ segs.
$3 \overline{\mathrm{MK}} \cong \overline{\mathrm{RK}}$
3 Given
$4 \overline{\mathrm{PK}} \cong \overline{\mathrm{PK}}$
4 Reflexive prop
$5 \triangle \mathrm{PRK} \cong \triangle \mathrm{PMK}$
5 SSS
$6 \angle P R K \cong \angle P M K$
6 CPCTC
$7 \angle 3$ supp to $\angle P R K$
7 If $2 \angle s$ form a st $\angle$, they are supp.
$8 \angle 4$ supp to $\angle$ PMK
8 Same as 7
$9 \angle 3 \cong \angle 4$
9 Supp of $\cong \angle$ s are $\cong$.

8 Given: $\angle \mathrm{AEB} \cong \angle \mathrm{DEC}$
$\overline{\mathrm{AE}} \cong \overline{\mathrm{DE}}$
$\angle A \cong \angle D$
Concl: $\overline{\mathrm{AC}} \cong \overline{\mathrm{BD}}$
$1 \angle \mathrm{AEB} \cong \angle \mathrm{DEC}$
$2 \overline{\mathrm{AE}} \cong \overline{\mathrm{DE}}$
$3 \angle A \cong \angle D \quad 3$ Given
$4 \triangle \mathrm{AEB} \cong \triangle \mathrm{DEC} \quad 4 \mathrm{ASA}$
$5 \overline{\mathrm{AB}} \cong \overline{\mathrm{CD}} \quad 5 \mathrm{CPCTC}$
$6 \overline{\mathrm{BC}} \cong \overline{\mathrm{BC}} \quad 6$ Reflexive prop
$7 \overline{\mathrm{AC}} \cong \overline{\mathrm{BD}}$


1 Given
2 Given

7 Addition prop

```
9 Given: \odot O
        NOG\cong}\angle\textrm{POG
    Concl: \vec{RO}}\mathrm{ bis }\angleNRP
    1\odot0
    2 \overline{ON}\cong\overline{OP}
    3\angleNOG\cong}\angle\textrm{POG
    4\angle1 is supp to }\angle\mathrm{ NOG.
    5 }\angle2\mathrm{ is supp to }\angle\mathrm{ POG.
    6 }11\cong\angle
    7 \overline{OR}\cong\overline{OR}
    8\triangleONR\cong\triangleOPR
    9 }\angle\textrm{NRO}\cong\angle\textrm{PRO
    10 \vec{RO}}\mathrm{ bis }\angleNRP
10 Given: }\overline{\textrm{AZ}}\cong\overline{\textrm{ZB}
        Z mdpt of \overline{XY}
        \angleAZX\cong\angleBZY
        XW}\cong\overline{YW
    Prove: }\overline{\textrm{AW}}\cong\overline{\textrm{BW}
    1 }\overline{\textrm{AZ}}\cong\overline{\textrm{ZB}
    2Z is mdpt of }\overline{XY
    3 \overline{XZ}}\cong\overline{ZY
    4\angleAZX\cong}\BZY
    5 \triangleAZX \cong\triangleBZY
    6 \overline{XW}\cong}\cong\overline{YW
    7 \overline{AX}\cong\overline{BY}
    8 \overline{AW}}\cong\overline{\textrm{BW}
    2 Radii of a \odot are \cong.
    Given
        4 If 2 }\angle\textrm{s}\mathrm{ form a st }\angle\mathrm{ , they are
    supp.
    Same as 4
    Supp of \cong\angles are \cong.
    Reflexive prop
    SAS
```



```
1 Given
2 Radii of a \(\odot\) are \(\cong\).
3 Given
4 If \(2 \angle \mathrm{~s}\) form a st \(\angle\), they are
supp.
5 Same as 4
6 Supp of \(\cong \angle \mathrm{s}\) are \(\cong\).
7 Reflexive prop
8 SAS
9 CPCTC
10 If a ray divides an \(\angle\) into \(2 \cong \angle \mathrm{~s}\), it bis the \(\angle\).
10 Given: \(\overline{\mathrm{AZ}} \cong \overline{\mathrm{ZB}}\) Z mdpt of \(\overline{X Y}\) \(\angle A Z X \cong \angle B Z Y\) \(\overline{\mathrm{XW}} \cong \overline{\mathrm{YW}}\)
Prove: \(\overline{\mathrm{AW}} \cong \overline{\mathrm{BW}}\)
\(1 \overline{\mathrm{AZ}} \cong \overline{\mathrm{ZB}}\)
2 Z is mdpt of \(\overline{\mathrm{XY}}\)
\(3 \overline{X Z} \cong \overline{Z Y}\)
\(4 \angle A Z X \cong \angle B Z Y\)
\(5 \triangle A Z X \cong \triangle B Z Y\)
\(6 \overline{\mathrm{XW}} \cong \overline{\mathrm{YW}}\)
\(7 \overline{\mathrm{AX}} \cong \overline{\mathrm{BY}}\)
\(8 \overline{\mathrm{AW}} \cong \overline{\mathrm{BW}}\)
```



```
1 Given
2 Given
3 A mdpt divides a seg into
\(2 \cong\) segs.
4 Given
5 SAS
6 Given
7 CPCTC
8 Subtraction prop
11 Given: \(\overrightarrow{\mathrm{DF}}\) bis \(\angle \mathrm{CDE}\).
\(\overrightarrow{\mathrm{EF}}\) bis \(\angle \mathrm{CED}\). G mdpt of \(\overline{\mathrm{DE}}\) \(\overline{\mathrm{DF}} \cong \overline{\mathrm{EF}}\)
Prove: \(\angle \mathrm{CDE} \cong \angle \mathrm{CED}\) \(1 \overrightarrow{\mathrm{DF}}\) bis \(\angle \mathrm{CDE}\).
1 Given
\(2 \angle 1 \cong \angle 2 \quad 2 \mathrm{~A}\) bis divides an \(\angle\) into \(2 \cong \angle \mathrm{~s}\).
\(3 \overline{\mathrm{EF}}\) bis \(\angle \mathrm{CED}\).
3 Given
\(4 \angle 3 \cong \angle 4\)
4 Same as 2
5 G mdpt of \(\overline{\mathrm{DE}}\) \(6 \overline{\mathrm{DG}} \cong \overline{\mathrm{GE}}\)
5 Given
6 A mdpt divides a seg into \(2 \cong\) segs.
\(7 \overline{\mathrm{DF}} \cong \overline{\mathrm{EF}}\)
7 Given
8 Draw \(\overline{\mathrm{FG}}\)
8 Two pts determine a seg.
9 Reflexive prop
\(9 \overline{\mathrm{FG}} \cong \overline{\mathrm{FG}}\)
\(10 \triangle \mathrm{DFG} \cong \triangle \mathrm{EFG}\)
10 SSS
```

$11 \angle 2 \cong \angle 4$
11 CPCTC
$12 \angle 1 \cong \angle 3 \quad 12$ Substitution prop
$13 \angle \mathrm{CDE} \cong \angle \mathrm{CED}$
13 Addition prop

12 Given: $\overline{\mathrm{AC}}$ is the alt to $\overline{\mathrm{BD}}$. $\overline{\mathrm{AC}}$ is a median. $\angle B A C$ comp to $\angle D$
Concl: $\angle$ DAC comp to $\angle B$
$1 \overline{\mathrm{AC}}$ is alt to $\overline{\mathrm{BD}}$. 1 Given
$2 \angle \mathrm{ACB}$ and $\angle \mathrm{ACD}$ are 2 An alt of a $\triangle$ forms $\mathrm{rt} \angle \mathrm{s}$
$\mathrm{rt} \angle \mathrm{s}$. with one of the sides.
$3 \angle \mathrm{ACB} \cong \angle \mathrm{ACD} \quad 3 \mathrm{Rt} \angle \mathrm{s}$ are $\cong$.
$4 \overline{\mathrm{AC}}$ is a median. 4 Given
$5 \overline{\mathrm{BC}} \cong \overline{\mathrm{CD}}$
5 A median of a $\Delta$ divides one side into $2 \cong$ segs.
$6 \overline{\mathrm{AC}} \cong \overline{\mathrm{AC}} \quad 6$ Reflexive prop
$7 \triangle \mathrm{ACB} \cong \triangle \mathrm{ACD} \quad 7 \mathrm{SAS}$
$8 \angle B A C \cong \angle D A C, \quad 8 \mathrm{CPCTC}$

$$
\angle D \cong \angle B
$$

$9 \angle B A C$ comp to $\angle D \quad 9$ Given
$10 \angle$ DAC comp to $\angle B \quad 10$ Substitution prop

13 At any point $(x, y)$ where $y=11$ or $y=1$
$14 \mathrm{OA}+\mathrm{AP}+\mathrm{OC}+\mathrm{CD}+\mathrm{OP}=$ Perimeter of $\triangle \mathrm{AOP}$.
Let $O C=x, D P=16-x, C D=x+2$
$\mathrm{OA}=\mathrm{OB}=\mathrm{OD}=2 \mathrm{x}+2, \mathrm{AP}=\mathrm{CP}=18$.
$(2 x+2)+(18)+(x)+(x+2)+(16-x)=80$

$$
3 x+38=80 \quad \text { OB }=2(14)+2=30
$$

$$
3 x=42 \quad B P=18
$$

$\mathrm{x}=14 \quad 30+18=48$
$\mathrm{OB}+\mathrm{BP}=48$

15 Given: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
$\overline{\mathrm{BD}} \cong \overline{\mathrm{CE}}$
Prove: $\angle 1 \cong \angle 2$

$1 \overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}, \overline{\mathrm{BD}} \cong \overline{\mathrm{CE}} \quad 1$ Given

| $2 \overline{\mathrm{AD}} \cong \overline{\mathrm{AE}}$ | 2 Addition prop |
| :--- | :--- |
| $3 \angle \mathrm{~A} \cong \angle \mathrm{~A}$ | 3 Reflexive prop |
| $4 \triangle \mathrm{ADC} \cong \triangle \mathrm{AEB}$ | 4 SAS |
| $5 \overline{\mathrm{DC}} \cong \overline{\mathrm{EB}}$ | 5 CPCTC |
| $6 \overline{\mathrm{BC}} \cong \overline{\mathrm{BC}}$ | 6 Reflexive prop |
| $7 \triangle \mathrm{DBC} \cong \triangle \mathrm{ECB}$ | 7 SSS |
| $8 \angle \mathrm{DBC} \cong \angle \mathrm{ECB}$ | 8 CPCTC |
| $9 \angle 1$ is supp to $\angle \mathrm{DBC}$, | 9 If sum of $2 \angle \mathrm{~s}$ is st $\angle, \angle \mathrm{s}$ |
| $\angle 2$, is supp to $\angle \mathrm{ECB}$. | $\quad$ are supp. |
| $10 \angle 1 \cong \angle 2$ | 10 Supps of $\cong \angle \mathrm{s}$ are $\cong$. |

## Answers to Isosceles $\triangle$ HW Day 6

20. Given: $\angle A$ is the vertex of an isosceles $\Delta$

The number of degrees in $\angle B$ is twice the number of centimeters in $\overline{B C}$

The number of degrees in $\angle \mathrm{C}$ is three times the number of centimeters is $\overline{A B}$

$$
\begin{aligned}
& m \angle B=x+6 \\
& m \angle C=2 x-54
\end{aligned}
$$

Find: The perimeter of $\triangle A B C$

$$
\begin{aligned}
& x+6=2 x-54 \\
& \Rightarrow x=60
\end{aligned}
$$



$$
\therefore A B=A C=\frac{2(60)-54}{3}=22
$$

and

Given: $\overline{C E} \cong \overline{C F}$
21.

$$
\Varangle F \cong \Varangle 3
$$

$\Varangle E$ is supp.to $\Varangle 5$
Prove: $\triangle C D G$ is isosceles


Statements

1. $\overline{C E} \cong \overline{C F}$
$\Varangle F \cong \Varangle 3$
$\Varangle E$ is supp.to $\Varangle 5$
2. $\Varangle E \cong \Varangle F$
3. $\Varangle E \cong \Varangle 3$
4. $\Varangle 4$ is supp. to $\Varangle 5$
5. $\Varangle E \cong \Varangle 4$
6. $\Varangle 3 \cong \Varangle 4$
7. $\overline{C D} \cong \overline{C G}$
8. $\triangle \mathrm{CDG}$ is Isosceles

Reasons

1. Given
2. If $\triangle$, then $\Delta$
3. Transitive Prop. (1, 2)
4. Linear Pair Thm
5. Congruent Suppl. Thm
6. Transitive Prop. $(3,5)$
7. If $\Delta \Delta$, then $\Delta$
8. Definition of Isosceles $\Delta$
9. Given: $\odot O, \odot P ; \overleftrightarrow{A B}$ bisects $\Varangle s O A P$ and $O B P$. Prove: Figure AOBP is equilateral.


## Statements

Reasons

1. $\odot O, \odot P ; \overleftrightarrow{A B}$ bisects $\Varangle S O A P$ and $O B P$.
(A) $2 . \Varangle 1 \cong \Varangle 2$
(A) $\Varangle 3 \cong \Varangle 4$
(S) 3. $\overline{A B} \cong \overline{A B}$
2. $\triangle A O B \cong \triangle A P B$
3. $\overline{A O} \cong \overline{A P}$
$\overline{B O} \cong \overline{B P}$
4. $\overline{A O} \cong \overline{B O}$

$$
\overline{A P} \cong \overline{B P}
$$

7. $\overline{A O} \cong \overline{B O} \cong \overline{A P} \cong \overline{B P}$
8. Figure $A O B P$ is equilateral
9. Given
10. Definition of angle bisector
11. Reflexive Property
12. $\operatorname{ASA}(2,3,2)$
13. СРСTC
14. All radii of a $\odot$ are $\cong$
15. Transitive Prop. $(5,6)$
16. If a figure has all sides $\cong \rightarrow$ equil.

24 Given: Figure XSTOW is equilateral and equiangular.
Prove: $\triangle Y T O$ is isos.


1 XSTOW is equilateral and equiangular.
(S) $2 \overline{S T}=\overline{W O}$
(S) $3 \overline{\mathrm{TO}} \cong \overline{\mathrm{TO}}$
(A) $4 \angle \mathrm{STO} \cong \angle W O T$
$5 \Delta S T O \cong \triangle W O T$
$6 \angle \mathrm{YOT} \approx \angle \mathrm{YTO}$
$7 \overline{\mathrm{TY}} \approx \overline{\mathrm{YO}}$
$8 \triangle \mathrm{YTO}$ is isos.

## 1 Given

2 If a figure is equilateral, all sides are $\equiv$.

3 Reflexive prop
4 If a figure is equiangular, all $\angle \mathrm{s}$ are $\equiv$.
$5 \operatorname{SAS}(2,4,3)$
6 CPCTC
7 If $\Delta \Delta$ then $\Delta$
8 If a $\Delta$ has at least 2 sides
$\equiv$, the $\Delta$ is isos.
25.

Given: $\triangle F E D$ is equilateral
Find: $\quad x, y$, and $m<F$


$$
6 y+12=3 x-6 \quad(x+y)+(3 x-6)=90
$$

$$
\Rightarrow 6 y=3 x-18 \quad \Rightarrow 4 x+y=96
$$

$$
\Rightarrow y=\frac{1}{2} x-3 \longrightarrow 4 x+\left(\frac{1}{2} x-3\right)=96
$$

$$
\Rightarrow \frac{8}{2} x+\frac{1}{2} x=99
$$

$$
\Rightarrow \frac{9}{2} x=99
$$

$$
\Rightarrow x=22
$$

$$
\Rightarrow y=\frac{1}{2}(22)-3=8
$$

$$
\therefore m \angle F=6(8)+12=60^{\circ}
$$

Given: $\overline{B E} \perp \overline{A D}, \overline{A C} \perp \overline{B D}$,

$$
\overline{A C} \cong \overline{B E}, \overline{D E} \cong \overline{E C}
$$

Prove: $\triangle \mathrm{DEC}$ is equilateral


Statements
(S) $1 . \overline{B E} \perp \overline{A D}, \overline{A C} \perp \overline{B D}$, $\overline{A C} \cong \overline{B E}, \overline{D E} \cong \overline{E C}$
2. $\Varangle 1$ and $\Varangle 2$ are $r t ~ \Varangle s$
(A) $3 . \npreceq 1 \cong \Varangle 2$
(A) $4 . \Varangle D \cong \Varangle D$
5. $\triangle D A C \cong \triangle D E B$
6. $\overline{D C} \cong \overline{D E}$
7. $\overline{D C} \cong \overline{D E} \cong \overline{E C}$
8. $\triangle D E C$ is equilateral

1. Given
2. Def of $\perp$ lines
3. all right $\Varangle s \cong$
4. Reflexive Property
5. AAS $(3,4,1)$
6. CPCTC
7. Transitive Prop. (1, 6)
8. If a figure has all sides $\cong \rightarrow$ equil.

Geometry Honors

## Answer Key

## Proving Triangles Congruent with Hypotenuse Leg

Page 158 \#'s 5 , 12 and 17


Right Angle Theorem and Equidistance Theorems
Pages 182-183 \#'s 4, 9, 14


Page 189-190 \#'s 14, 15, 16, 17, and 20


## Answers to Detour Proofs

Detour Proofs pages 174-175 \#'s 11, 13, 14, 17


Page 141


Answers to Missing Diagram Proofs Page 179 \#8, 11, 12, 14
б.

If the median to a side of a $\Delta$ is also an altitude to that side, then the $\Delta$ is isosceles.

Given: $\overline{\mathrm{BD}}$ is a median $\overline{B D}$ is an altitude

Prove: $\quad \triangle A B C$ is isosceles


Statements

1. $\overline{B D}$ is a median
2. $D$ is the midpoint of $\overline{C A}$

S 3. $\overline{\mathrm{AD}} \cong \overline{\mathrm{DC}}$
4. $\overline{B D}$ is an altitude
5. $\overline{B D} \perp \overline{C A}$
6. $\angle$ BDA and $\angle B D C$ are right $\angle S$

A 7. $\angle B D A \cong \angle B D C$
S 8. $\overline{B D} \cong \overline{B D}$
9. $\triangle A B D \cong \triangle C B D$
10. $\overline{B A} \cong \overline{B C}$
11. $\triangle A B C$ is isosceles

1. Given
2. Definition of Median
3. Definition of Midpoint
4. Given
5. Defn. of Altitude
6. Defn. of $\perp$ Segments
7. All Right Angles are Congruent
8. Reflexive Property
9. $\operatorname{SAS}(2,5,6)$
10. СРСTC
11. Definition of Isosceles $\Delta$

Prove that if $2 \Delta s$ are $\simeq$, then any pair of corresponding medians are $\simeq$.

Given: $\quad \triangle A B C \cong \triangle X Y Z$
$\overline{\mathrm{AD}} \& \overline{\mathrm{XW}}$ are medians

Prove: $\quad \overline{\mathrm{AD}} \cong \overline{\mathrm{XW}}$


Statements
Statements
Reasons

1. $\triangle A B C \cong \triangle X Y Z$

S 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{XY}}$
A 3. $\angle B \cong \angle Y$
4. $\overline{B C} \cong \overline{Y Z}$
5. $\overline{\mathrm{AD}} \& \overline{\mathrm{XW}}$ are medians
6. $\mathrm{D} \& \mathrm{~W}$ are midpoints
7. $\overline{B D} \cong \frac{1}{2} \overline{B C}, \overline{Y W} \cong \frac{1}{2} \overline{Y Z}$

S
8. $\overline{B D} \cong \overline{Y W}$
9. $\triangle A B D \cong \triangle X Y W$
10. $\overline{\mathrm{AD}} \cong \overline{\mathrm{XW}}$

1. Given
2. CPCTC
3. СРСТС
4. СРСТС
5. Given
6. Definition of Median of $\Delta$
7. Definition of Midpoint
8. Division Property of $\cong$ Segments
9. SAS $(2,3,8)$
10. СРСТС
11. 

Prove that if a $\Delta$ is isosceles, then the $\Delta$ formed by its base and the $\angle$ bisectors of its base $\angle 5$ is also isosceles.

Given: $\quad \triangle A B C$ is isosceles with vertex $\angle A B C$ $\overline{\mathrm{AD}} \& \overline{\mathrm{CD}}$ are $\angle$ bisectors

Prove: $\quad \triangle A D C$ is isosceles


Statements
Reasons

1. $\triangle A B C$ is isos, with vertex $\angle A B C$
2. $\overline{A B}=\overline{C B}$
3. $\angle B A C \cong \angle B C A$
4. $\overline{\mathrm{AD}} \& \overline{\mathrm{CD}}$ are $\angle$ bisectors
5. $\angle D A C \cong \frac{1}{2} \angle B A C, \angle D C A \cong \frac{1}{2} \angle B C A$
6. $\angle \mathrm{DAC}=\angle \mathrm{DCA}$
7. $\overline{A D}=\overline{C D}$
8. $\triangle A D C$ is isosceles
9. Given
10. Definition of Legs of Isosceles $\Delta$
11. If $\Delta$, then $\Delta$
12. Given
13. Definition of Angle Bisector
14. Division Property of = Angles
15. If $\Delta \Delta$, then $\Delta$
16. Definition of Isosceles $\Delta$
17. 

Prove that if a point on the base of an isos. $\Delta$ is equidistant from the midpoints of the legs, then that point is the midpoint of the base.
Given: $\quad \triangle A B C$ is isos. with vertex $\angle B A C$
D \& E are midpoints
$F$ is equidistant from $D \& E$
Prove: $\quad \mathrm{F}$ is the midpt. of $\overline{\mathrm{BC}}$


Statements
Reasons

1. $F$ is equidistant from $D \& E$
$S$ 2. $\overline{F D} \cong \overline{F E}$
2. $\triangle A B C$ is isos. with vertex $\angle B A C$

S 4. $\overline{A B} \cong \overline{A C}$
5. $D \& E$ are midpoints
6. $\overline{A D} \cong \frac{1}{2} \overline{A B}, \overline{A E} \cong \frac{1}{2} \overline{A C}$
s
7. $\overline{\mathrm{AD}} \cong \overline{\mathrm{AE}}$
8. Draw $\overline{\mathrm{AF}}$

SS 9. $\overline{\mathrm{AF}} \cong \overline{\mathrm{AF}}$
10. $\triangle A D F \cong \triangle A E F$

A 11. $\angle \mathrm{DAF} \cong \angle E A F$
12. $\triangle A B F \cong \triangle A C F$
13. $\overline{\mathrm{BF}} \cong \overline{\mathrm{FC}}$
14. $F$ is the midpoint of $\overline{B C}$

1. Given
2. Definition of Equidistant
3. Given
4. Definition of Isosceles $\Delta$
5. Given
6. Definition of Midpoint
7. Division Property of $\simeq$ Segments
8. 2 points determine a line
9. Reflexive Property
10. SSS $(2,7,9)$
11. CPCTC (from 10)
12. SAS $(4,11,9)$
13. CPCTC (from 12).
14. Definition of Midpoint

[^0]:    1
    2
    3

    4

    5

